Defining ”wiper fibration”

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Working with a new set of technologies, it sometimes happen in real world that we may better employ an essentially conceptual portion of a mathematical theory, in order to solve practical task in a business.

Today we introduce a type of (Grothendieck) fibration equipped with G-action that we named ”wiper fibration”, obtained out of the theoretical line, despite that it first came across when we were brain storming over the neural network model.

Definition. Let M be a real n-manifold and $PT(M)$ its associated projective bundle of the tangent bundle, where the fiber is homeomorphic to $\mathbb{R}P^{n-1}$. Suppose there given a (not necessarily continuous) function $\mu : M \times \mathbb{R}P^{n-1} \to [0, \infty)$ of finite value and a topological group G that acts freely and transitively on $\mathbb{R}P^{n-1}$.¡/p¿¡p¿The subspace E of $PT(M)$ is given by restricting fibers as:

$$E = \{(x, v) \in M \times \mathbb{R}P^{n-1} : v = \sup \mu(x, \mathbb{R}P^{n-1})\},$$

and we call the data (E, M, G, μ) together with the canonical projection $E \to M$, the wiper fibration.

Proposition 1. wiper fibration (E, M, G, μ) is a Grothendieck fibration on a discrete category of sets if for any $(x, v) \in E$ and $y \in B$, there exists the unique $g \in G$ such that $gv \in p^{-1}(y)$.

Proof. For any $(x, v) \in E$ and a function f such that $f(y) = x$, we have the unique function $\phi : E \to E$ such that $\phi(y, v) = (f(y), g_v v)$, where $g_v \in G$ can be uniquely chosen by assumption. □

Restricting attention to a compact subset B of $\mathbb{R}^2$, it may be easily imagined that the corresponding wiper fibration well depicts the coordinate and direction of windscreen wiper on which the maximum amount of rain drops are loaded.

For a more practical use, the G action and the condition of μ should be refined in the following way. The choice of $g \in G$ may better to require a sense of continuity so it still remains fibration over a topological category. The function μ wants to be consistent so the fiber behaves well.